Physics-Informed Neural Network For Prediction of 3D Projectile Motion

Application To Table-Tennis Sport

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- Helps professionals improve sportsmen **performance** and reduce the **risk of injuries**.
- **Expensive**, **intrusive** and time consuming system.
- Confined to laboratory experiments.

Figure – Marker-based activity analysis

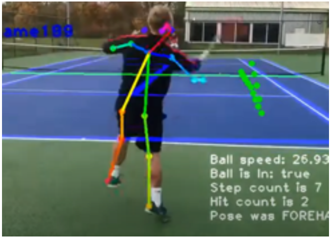


Figure – Marker-less activity analysis

Objective :

- **Computer vision** method allowing the study of **human activities** and interaction with its **physical environment**.
- Estimation of **3D kinematics** using **monocular** vision.
- Application of the method on **Table-Tennis**.

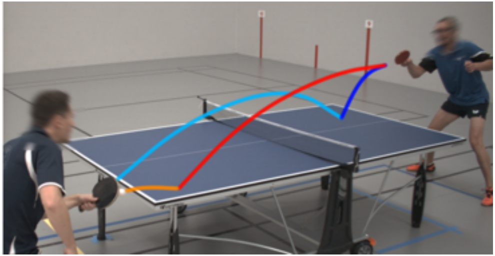


Figure – French table tennis championships 2024.

- Creating an efficient and **robust** model of the ball's **trajectory** is essential to understand the effects applied on the ball.
- Studying the **3D kinemtaics** of **small objects** presents a significant challenge in terms of 3D modeling.
- To achieve this, we develop a Physics-Informed Neural Networks (**PINNs**) to predict 3D ball **trajectories** and infer non-observable **kinematics** parameters.

Newton's second law

$$m\ddot{\mathbf{s}} = m\frac{d\dot{\mathbf{s}}}{dt} = \mathbf{F}_G + \mathbf{F}_D + \mathbf{F}_L \quad (1)$$

given the ball position $\mathbf{s}$, here $\ddot{\mathbf{s}}$ is the acceleration and $\dot{\mathbf{s}}$ is the ball velocity $\mathbf{V}$, and m is the mass of the ball. The gravitational force $\mathbf{F}_G$, $\mathbf{F_D}$ is the drag force, $\mathbf{F_L}$ is the lift force.

$$F_D = \|\mathbf{F}_D\| = -\frac{1}{2}C_D\rho AV^2 \qquad F_L = \|\mathbf{F}_L\| = \frac{1}{2}C_L(t)\rho AV^2$$

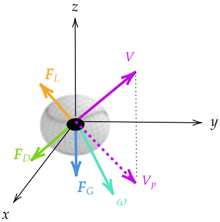


Figure – Schematic diagram showing forces applied on a ball. V the ball velocity and ω represent the rotation velocity.

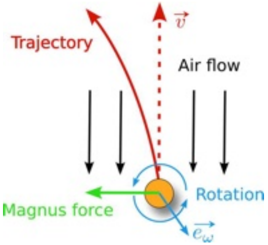
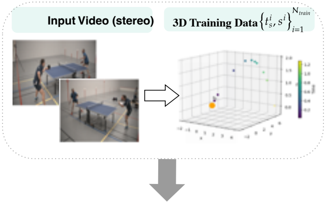


Figure – Impact of Magnus effect on ball flight.

PINNS for Trajectory Prediction and Kinematics Estimation



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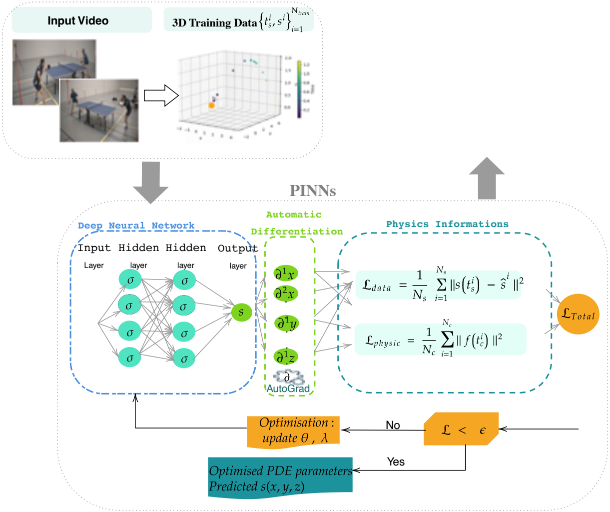


PINNS for Trajectory Prediction and Kinematics Estimation

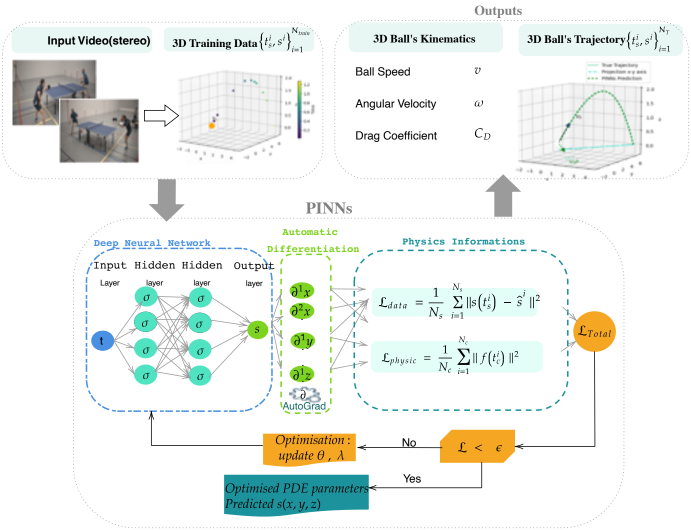


Figure – Schematic diagram of the framework used in this work.

PINNs [2] are comprised of three key components :

- **Feed-forward network** : aims to predict the position $\hat{s}(x, y, z)$ at any t.
- **Physics Constraints**, the derivatives of $\hat{s}(x, y, z)$ with respect to the collocation points $\{t^i_{phy}\}$ are calculated using automatic differentiation(AD)[2].

Residual Functions

$$\begin{aligned} f_x &= -\ddot{x} - k \times V\left\{(C_D\dot{x}) - \left(\frac{C_L}{\omega}(\omega_y\dot{z} - \omega_z\dot{y})\right)\right\} \\ f_y &= -\ddot{y} - k \times V\left\{(C_D\dot{y}) - \left(\frac{C_L}{\omega}(\omega_z\dot{x} - \omega_x\dot{z})\right)\right\} \\ f_z &= -\ddot{z} - g - k \times V\left\{(C_D\dot{z}) - \left(\frac{C_L}{\omega}(\omega_x\dot{y} - \omega_y\dot{x})\right)\right\} \end{aligned} \tag{2}$$

- **Optimization mechanism**, to minimize the residual functions over the domain of interest, while simultaneously satisfying the governing ODEs and any available data.

Optimization Mechanism

$$(\theta^*, \lambda^*) = \arg\min_{\theta,\lambda}(\beta_{phy}\mathcal{L}_f + (1-\beta_{phy})\mathcal{L}_s) \quad (3)$$

here,$\theta = \{weights, biases\}$, $\lambda = \{C_D, \omega_x, \omega_y, \omega_z\}$, $\mathcal{L}_s$ is the loss with respect to the data, $\mathcal{L}_f$ is the loss with respect to the physic constraints, and $\beta_{phy} \in [0, 1]$ is the relative weight of the physics loss [3].Loss functions, are calculated using mean squared error [5].

$$\begin{aligned} \mathcal{L}_s = MSE_s &= \frac{1}{N_d}\sum_{i=1}^{N_d}\left\|s\left(t_s^i\right) - \hat{s}^i\right\|^2 \\ \mathcal{L}_f = MSE_f &= \frac{1}{N_c}\sum_{i=1}^{N_p}\|f(t_{phy}^i)\|^2 \end{aligned} \quad (4)$$

where $\left\{t_s^i, s^i\right\}_{i=1}^{N_d}$, denote the training data and $\{t_{phy}^i\}_{i=1}^{N_p}$ are the physic collocation points.

Simulation of Table-Tennis Ball Trajectories

- Initial setting related to the rotation vector is used to simulate non-planar trajectories.
- Several stroke classes are simulated by solving the ODEs using Runge-Kutta method [4].
- 100 trajectories for each stroke type : Topspin, Push and Counter Attack.

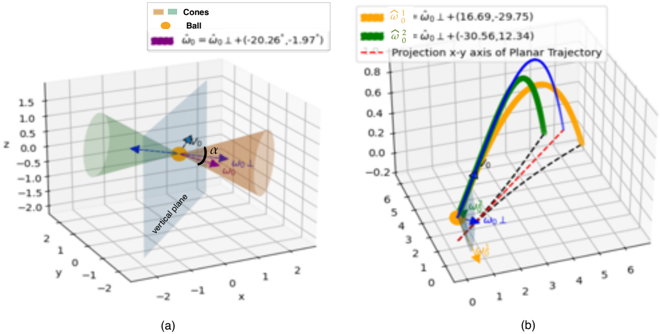
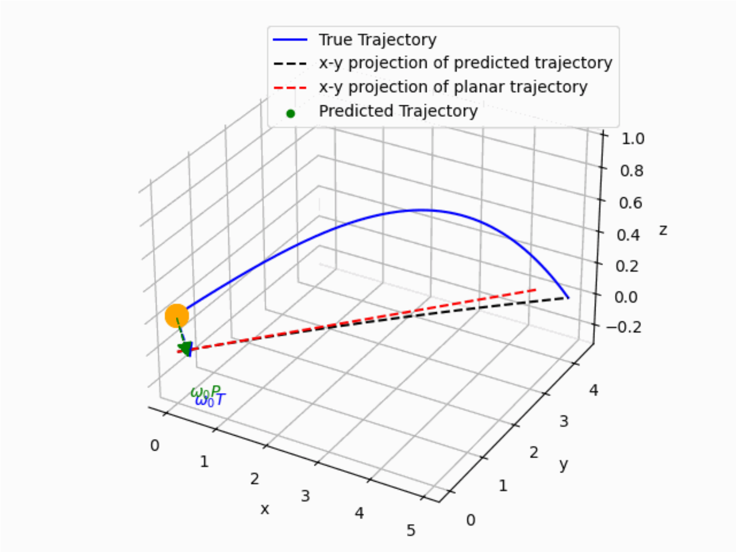
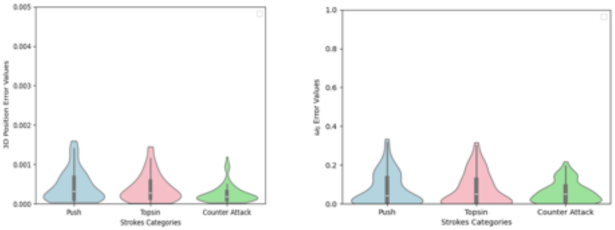


Figure – (a)Illustration of the method for setting the initial conditions related to the vectors V_0 and ω_0. (b) Example of 3 Topspin simulated trajectories.

Training and Evaluation of PINNs





(a) (b)

Figure – (a) Violin plots for relative error values of PINNs 3D position predictions. (b) Violin plots for realtive error values of PINNs ω_0 estimations.

Table – Comparison of methods in term of the average estimated relative error of the rotation and translation speed for all the strokes classes.

Method	Rotation Speed $\mathcal{E}_{\omega_0}$	Translation Speed $\mathcal{E}_{V_0}$
Calandre et *al.*[1]	0.41	3.04
PINNS	**0.07**	**0.00**

PINNS Validation on Real Trajectories Data

- Dataset : 33 Video sequences include several strokes from Table-Tennis gameplay.
- Annotated based on three stroke categories : Top Spin, Counter Attack, and Push

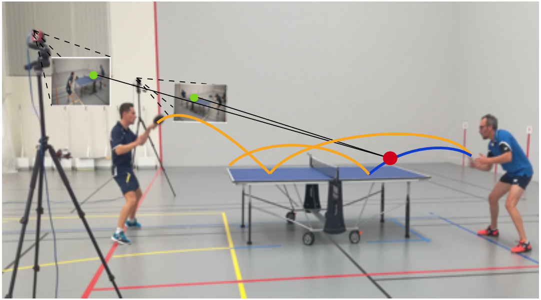
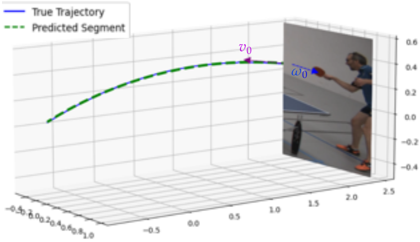
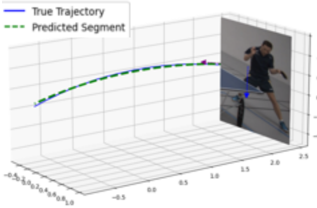


Figure – Frame from video sequences of Table-Tennis. Illustrate 3D reconstruction of ball using stereo-vision.

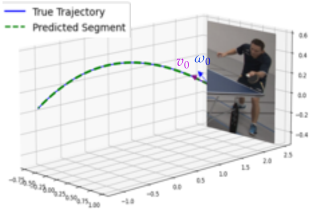
PINNS Validation on Real Trajectories Data



(a) Counter Attack



(b) Topspin



(c) Push

Figure – Results of the PINNS prediction of a segment from the real trajectories dataset for different stroke classes.

Table – Average estimated relative error of the 3D position and translation speed for each of the three stroke types.

Stroke	Position $\mathcal{E}_p$	Translation Speed $\mathcal{E}_{V_0}$
Topspin	0.003	0.064
Push	0.013	0.067
Counter	0.005	0.093

Table – Average estimated values of ω_0, V_0 and C_d for each of the three stroke types.

(ω_0 in rps and V_0 in m/s and Cd is dimensionless)

Stroke	Rotation Speed ω_0	Translation Speed V_0	Drag Coefficient C_d
Topspin	52.38	14.18	0.38
Push	-28.07	6.73	0.53
Counter Attack	24.78	19.96	0.40

- Linear Support Vector Machine (**SVM**) classification is applied on all the 33 real sequences.
- The estimated ω_0 and V_0 are **relevant features** and can be used to classify the type of a given stroke.

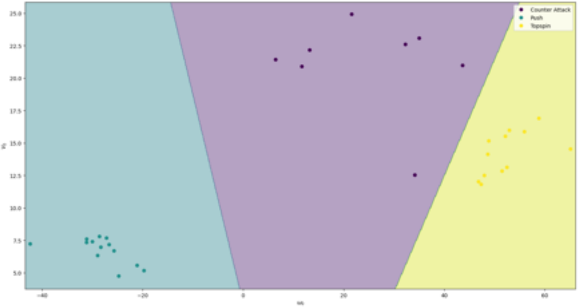


Figure – Classification of data points with respect to the ω_0 and V_0 values.

Conclusion and Perspectives

- Using small amount of **simulated data** points and the **physics constraints** we show that our model is able to predict the **3D position** and **3D kinematics** of both **planar and non-planar trajectories**.
- PINNS **improve** the estimation of initial **translation and rotation speed** comparing with other method.
- The model **perform** the task and shows **accurate prediction** with the **real dataset** of Table-tennis 3D trajectories obtained using **stereovision**.
- Our future aim is to further explore the use of 3D data trajectories reconstructed using a single camera (**monovision**)

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Thank You

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